Trigonometric Functions and Identities

Trigonometric Functions

Trigonometric Functions	$\sin\theta = \frac{\text{opp}}{\text{hyp}}$	$\cos\theta = \frac{\text{adj}}{\text{hyp}}$	$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{\sin\theta}{\cos\theta}$
	$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{1}{\sin\theta}$	$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{1}{\cos\theta}$	$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{\cos\theta}{\sin\theta}$
Law of Cosines	$a^2 = b^2 + c^2 - 2bc\cos A$	$b^2 = a^2 + c^2 - 2ac\cos B$	$c^2 = a^2 + b^2 - 2ab\cos C$
Law of Sines	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$	**Heron's Formula**	$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$
Linear Speed	$v = \frac{s}{t}$	**Angular Speed**	$\omega = \frac{\theta}{t}$

Trigonometric Identities

Reciprocal	$\sin\theta = \frac{1}{\csc\theta}$	$\cos\theta = \frac{1}{\sec\theta}$	$\tan\theta = \frac{1}{\cot\theta}$
	$\csc\theta = \frac{1}{\sin\theta}$	$\sec\theta = \frac{1}{\cos\theta}$	$\cot\theta = \frac{1}{\tan\theta}$
Pythagorean	$\sin^2\theta + \cos^2\theta = 1$	$\tan^2\theta + 1 = \sec^2\theta$	$\cot^2\theta + 1 = \csc^2\theta$
Cofunction	$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$	$\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$	$\sec\theta = \csc\left(\frac{\pi}{2} - \theta\right)$
	$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$	$\cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$	$\csc\theta = \sec\left(\frac{\pi}{2} - \theta\right)$
Odd-Even	$\sin(-\theta) = -\sin\theta$	$\cos(-\theta) = \cos\theta$	$\tan(-\theta) = -\tan\theta$
	$\csc(-\theta) = -\csc\theta$	$\sec(-\theta) = \sec\theta$	$\cot(-\theta) = -\cot\theta$

Sum & Difference

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ | $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ | $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$ | $\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

Double-Angle

$\cos 2\theta = \cos^2\theta - \sin^2\theta$ | $\cos 2\theta = 2\cos^2\theta - 1$ | $\cos 2\theta = 1 - 2\sin^2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$ | $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

Power-Reducing

$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$ | $\cos^2\theta = \frac{1 + \cos 2\theta}{2}$ | $\tan^2\theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$

Half-Angle

$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{2}}$ | $\cos\frac{\theta}{2} = \pm\sqrt{\frac{1 + \cos\theta}{2}}$

$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}}$ | $\tan\frac{\theta}{2} = \frac{1 - \cos\theta}{\sin\theta}$ | $\tan\frac{\theta}{2} = \frac{\sin\theta}{1 + \cos\theta}$

Product-to-Sum

$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$ | $\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$

$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$ | $\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$

Sum-to-Product

$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$ | $\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$

$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$ | $\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$